Problem I: *(30 points)* 28

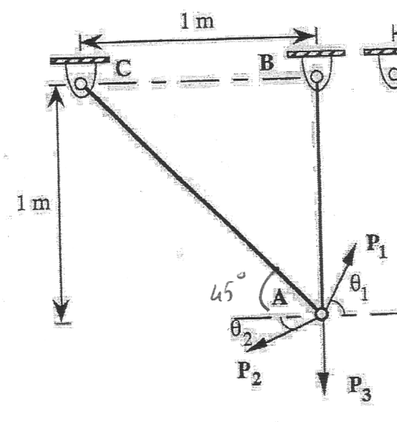


Figure I-a

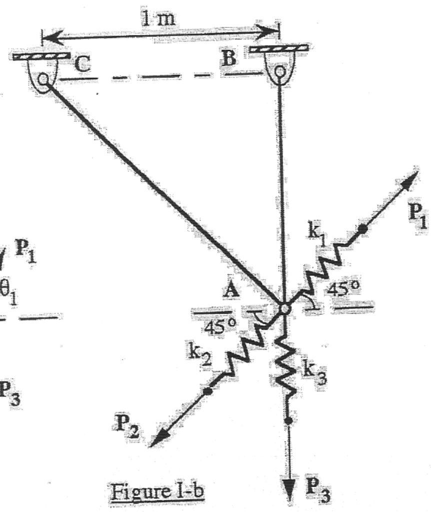
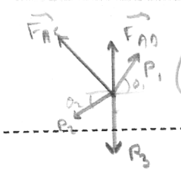


Figure I-b

Tick Boxes to check that you solved all questions

The cable system (AB and AC) is used to support the forces shown in Figures I-a and I-b.

1. Referring to Figure I-a, let $\theta_1=60^\circ$ and $\theta_2=30^\circ$.
   If a tensile force of 50 N is measured in cable AB, calculate P_3 for P_1=200 N and P_2=100 N. What is the force in cable AC. *(8 points)* ☑
   Calculate the force in cable AC if P_2=200 N while P_1 is kept at 200 N. Comment briefly on the result. *(7 points)* ☑



3

Calculations and/or Diagrams:

$\vec{F}_{AB} = 50\,N$ $P_1 = 200\,N$ $P_2 = 100\,N$

At equilibrium: $\Sigma \vec{F} = \vec{0}$

$+\uparrow \Sigma F_y = 0$ $50 + P_1 \sin\theta_1 - P_2 \sin\theta_2 - P_3 + F_{AC} \sin 45 = 0$ 2

$50 + 200 \sin 60 - 100 \sin 30 - P_3 + F_{AC} \sin 45 = 0$

$P_3 = 50 + 200 \sin 60 - 100 \sin 30 + F_{AC} \sin 45$

$P_3 = 173.2 + 0.71 F_{AC}$

$\xrightarrow{+} F_x = 0$

$P_1 \cos\theta_1 - F_{AC} \cos 45 - P_2 \cos\theta_2 = 0$ 2

Calculations and/or Diagrams (cont'd):

$200 \cos 60 - F_{AC} \cos 45 - 100 \cos 30 = 0$

$F_{AC} \cos 45 = 200 \cos 60 - 100 \cos 30$

$\boxed{F_{AC} = 18.95 N}$ ✓ (1)

$P_3 = 173.2 + 0.71 F_{AC}$

$\boxed{P_3 = 186.6 N}$ ✓ (1)

* If $P_2 = 200N$

$\pm \sum F_x = 0$ (2)

$P_1 \cos \theta_1 - P_2 \cos \theta_2 - F_{AC} \cos 45 = 0$

$200 \cos 60 - 100 \cos 30 - F_{AC} \cos 45 = 0$

$\boxed{F_{AC} = -103.5 N}$ ✓ (1)

The result is negative, hence, the force has a direction from C to A.
This is the opposite direction than that of the first case ? compression (1)

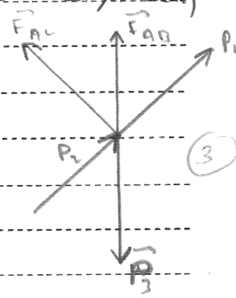
2. The system is modified with three springs of stiffness $k_1=k_2=k_3=1{,}000$ N/m and initial length $l_0=0.2$ m for all springs, as shown in Figure I-b. The final lengths of the springs, after the forces are applied, have been measured as $l_1=0.25$ m, $l_2=0.15$ m, and $l_3=0.4$ m. Calculate the forces in cables AB and AC. *(15 points)*

Calculations and Diagrams:

$P_1 = k_1|\Delta l| = k_1\{l_1 - l_0\} = 50\text{ N}$ 2

$P_2 = k_2|\Delta l_2| = k_2\{l_2 - l_0\} = \mp 50\text{ N}$ ✗ (it is a compression) 2

$P_3 = k_3|\Delta l_3| = k_3\{l_3 - l_0\} = 200\text{ N}$ 2



3

At equilibrium: $\Sigma\vec{F} = \vec{0}$

$\xrightarrow{+} F_x = 0$ 2

$P_1\cos 45 + P_2\cos 45 - F_{AC}\cos 45 = 0$

$P_1 + P_2 - F_{AC} = 0$

$\boxed{F_{AC} = 100\text{N}}$ ✓ 1

$+\uparrow F_y = 0$

$P_1\sin 45 + F_{AB} + P_2\sin 45 - P_3 \pm F_{AC}\sin 45 = 0$ 2

$F_{AB} = -P_2\sin 45 + P_3 - P_1\sin 45 - F_{AC}\sin 45$

$\cancel{\boxed{F_{AB} = 200\text{N}}}$

$= -50\sin 45 + 200 - 50\sin 45 - 100\sin 45$

$\boxed{F_{AB} = 58.6\text{ N}}$ ✓ 1

Problem II: *(15 points)* 15

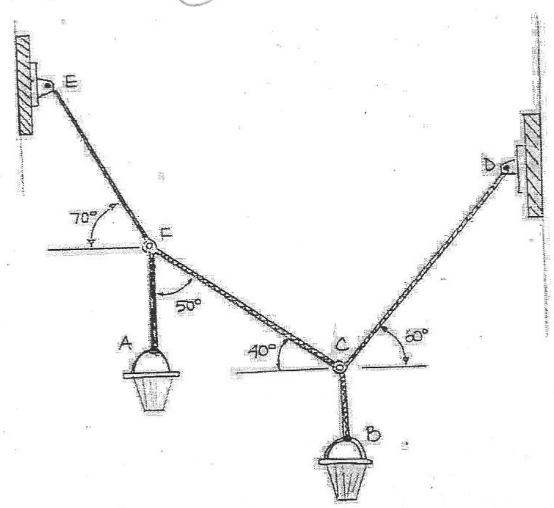
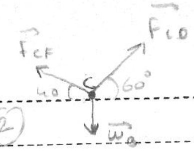


Figure II

The cords (or cables) suspend the buckets at A and B in the equilibrium position shown in Figure II. If the weight of the bucket at B is 40 lbs, determine the weight of the bucket at A. *(15 points)*

Calculations and/or Diagrams:



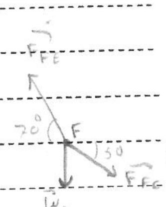
2

Equilibrium at C : $\sum \vec{F} = \vec{0}$

$\rightarrow F_x = 0$: $\begin{cases} F_{CD}\cos 60 - F_{CF}\cos 40 = 0 \\ F_{CD}\sin 60 + F_{CF}\sin 40 = 40 \end{cases}$ 2

$+\uparrow F_y = 0$ 2

$\boxed{F_{CD} = 31\,N \quad F_{CF} = 20.3\,N}$ ✓ 2



2

Equilibrium at F : $\sum \vec{F} = \vec{0}$ ✓ 2

$\rightarrow F_x = 0 \quad F_{FC}\sin 50 = F_{FE}\cos 70$

$F_{FE} = \dfrac{F_{FC}\sin 50}{\cos 70} = 45.5\,N$ ✓ 2

$+\uparrow F_y = 0 \quad F_{FE}\sin 70 - W_A - F_{FC}\cos 50 = 0$

$W_A = F_{FE}\sin 70 - F_{FC}\cos 50$

$\boxed{W_A = 29.7\,lbs}$ ✓

Problem III: *(25 points)* 23

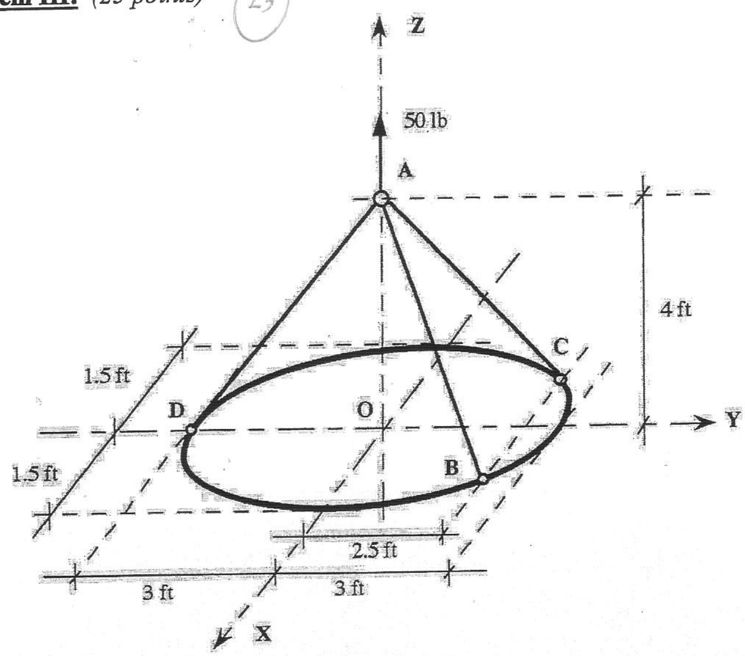


Figure III

An elliptical ring lying in the horizontal plane (X,Y) is supported by three cables AB, AC, and AD, as shown in Figure III. The equation of the ellipse for the ring is given as:

$$(X/a)^2 + (Y/b)^2 = 1$$

where a=1.5 and b=3.
Determine the forces in the cables if the ring has a weight of 50 lbs. *(25 points)*

F.B.D ?

Calculations and/or Diagrams:

1

A (0, 0, 4)

for y = 2.5 , $\left(\frac{x}{1.5}\right)^2 + \left(\frac{2.5}{3}\right)^2 = 1$

$\left(\frac{x}{1.5}\right)^2 = 0.31$

3

$\frac{x}{1.5} = 0.55$

$\boxed{x = 0.83}$ B(0.83, 2.5, 0)

C(-0.83, 2.5, 0) D(0, -3, 0)

Calculations and/or Diagrams (cont'd):

$\vec{AB} = 0.83\vec{i} + 2.5\vec{j} - 4\vec{k}$ $AB = 4.8$

$\vec{u}_{AB} = 0.17\vec{i} + 0.52\vec{j} - 0.84\vec{k}$

$\vec{AC} = -0.83\vec{i} + 2.5\vec{j} - 4\vec{k}$ $AC = 4.8$

$\vec{u}_{AC} = -0.17\vec{i} + 0.52\vec{j} - 0.84\vec{k}$

$\vec{AD} = -3\vec{j} - 4\vec{k}$ $AD = 5$

$\vec{u}_{AD} = -0.6\vec{j} - 0.8\vec{k}$

$\vec{F}_{AB} = F_{AB}\vec{u}_{AB}$ 1 $\vec{F}_{AC} = F_{AC}\vec{u}_{AC}$ 1 $\vec{F}_{AD} = F_{AD}\vec{u}_{AC}$ 1

At equilibrium: 2 2

$50\vec{k} + F_{AB}(0.17\vec{i} + 0.52\vec{j} - 0.84\vec{k}) + F_{AC}(-0.17\vec{i} + 0.52\vec{j} - 0.84\vec{k})$
$+ F_{AD}(-0.6\vec{j} - 0.8\vec{k}) = 0$

2

$0.17F_{AB} - 0.17F_{AC} = 0 \Rightarrow F_{AB} = F_{AC}$

$0.52F_{AB} + 0.52F_{AC} - 0.6F_{AD} = 0$ 6

$\Rightarrow 1.04F_{AB} = 0.6F_{AD} \Rightarrow F_{AD} = 1.73F_{AB}$

$-0.84(F_{AB} + 0.84F_{AC} - 0.8F_{AD}) = 50$

$-1.68F_{AB}$ $-1.384F_{AD} =$ -50

$-1.68F_{AB}$ -1.384 $F_{AB} = -50$

-3.064 $F_{AB} = -50$

$\vec{F}_{AB} = 2.8\vec{i} + 8.5\vec{j} - 13.7\vec{k}$

$\vec{F}_{AC} = -2.8\vec{i} + 8.5\vec{j} - 13.7\vec{k}$

~~$\vec{F}_{AD}$ ... $\vec{j}$ ... $22.6\vec{k}$~~

$\vec{F}_{AD} = -16.94\vec{j} - 22.6\vec{k}$

$\boxed{F_{AB} = 16.32 \text{ lbs} \quad F_{AC} = 16.32 \text{ lbs} \quad F_{AD} = 28.23 \text{ lbs}}$

4

Problem IV: *(20 points)* 20

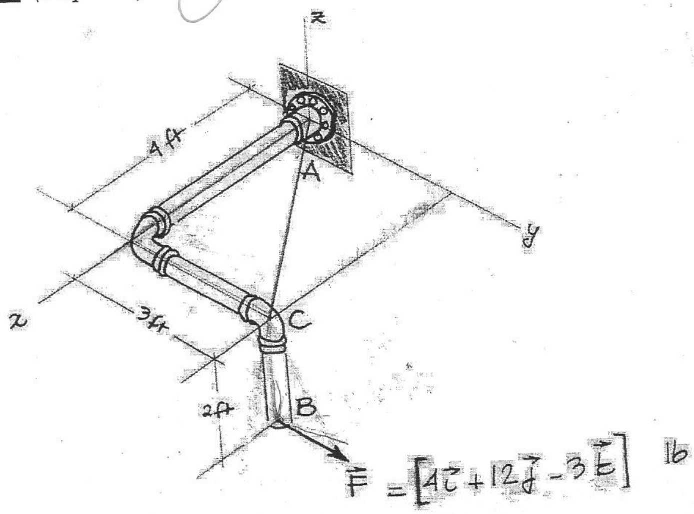


Figure IV

Referring to Figure IV, determine the moments of the force vector **F** about the x, y, and z axes. Solve the problem using: (a) a Cartesian vector approach and (b) a Scalar approach. Express each result as a Cartesian vector. *(14 points)*

Determine the moment of the force vector **F** about an axis extending between points A and C. Express the result as a Cartesian vector. *(6 points)*

Calculations and/or Diagrams:

(a) $\vec{F} = 4\vec{i} + 12\vec{j} - 3\vec{k}$

$\vec{AB} = \vec{r} = 4\vec{i} + 3\vec{j} - 2\vec{k}$

$M_x = \begin{vmatrix} 1 & 0 & 0 \\ 4 & 3 & -2 \\ 4 & 12 & -3 \end{vmatrix} = 15$ $\vec{M}_x = \{15\vec{i}\}$ lb.ft

$M_y = \begin{vmatrix} 0 & 1 & 0 \\ 4 & 3 & -2 \\ 4 & 12 & -3 \end{vmatrix} = +4$ $\vec{M}_y = \{+4\vec{j}\}$ lb.ft

$M_z = \begin{vmatrix} 0 & 0 & 1 \\ 4 & 3 & -2 \\ 4 & 12 & -3 \end{vmatrix} = 36$ $\vec{M}_z = \{36\vec{k}\}$ lb.ft

8

Calculations and/or Diagrams (cont'd):

(b) $F_x = 4\ lb$ $F_y = 12\ lb$ $F_z = -3\ lb$

$M_x = F_y z - F_z y$

$= 12 \times 2 - 3 \times 3$

$= 24 - 9$

$= 15$ $\vec{M}_x = \{15\vec{i}\}\ lb.ft$

$M_y = -F_x z + F_z x$

$= -4 \times 2 + 3 \times 4$

$= 4$ $\vec{M}_y = \{+4\vec{j}\}\ lb.ft$

6

$M_z = F_y x - F_x y$

$= 12 \times 4 - 4 \times 3$

$= 36$ $\vec{M}_z = \{36\vec{k}\}\ lb.ft$

$\vec{AC} \{ = 4\vec{i} + 3\vec{j}$ $AC = 5$

$\vec{u}_{AC} = 0.8\vec{i} + 0.6\vec{j}$ 2

$\vec{r} = \vec{AB} = 4\vec{i} + 3\vec{j} - 2\vec{k}$ $M_{AC} = \vec{u}_{AC} \cdot (\vec{r} \times \vec{F})$

$$M_{AC} = \begin{vmatrix} 0.8 & 0.6 & 0 \\ 4 & 3 & -2 \\ 4 & 12 & -3 \end{vmatrix} = 0.8(15) - 0.6(-4)$$

$= 14.4\ lb.ft$ 2

$\vec{M}_{AC} = M_{AC}\, \vec{u}_{AC}$

$= 14.4(0.8\vec{i} + 0.6\vec{j})$ 2

$= \{11.5\vec{i} + 8.6\vec{j}\}\ lb.ft$

Problem V: *(10 points)*

10

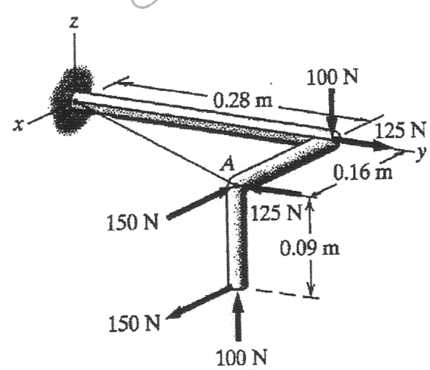


Figure V

Three couples are applied to the bent bar shown in Figure V. Determine the vector components of the resultant moment about x, y, and z, and express as a Cartesian vector. *(10 points)*
(Hint: This a 5 mn question maximum. Take advantage of the notion of couples)

Calculations and/or Diagrams:

$+\circlearrowleft M_x = 0$ $\boxed{\vec{M_x} = \vec{0}}$ $M_R = \Sigma M_c$

$+\circlearrowleft M_y = -100 \times 0.16 - 150 \times 0.09$ $M_{Rx} = \Sigma M_{cx}$

$= -29.5$ N.m $M_{Ry} = \Sigma M_{cy}$

$\vec{M_y} = \{-29.5\,\vec{j}\}$ N.m $M_{Rz} = \Sigma M_{cz}$

$+\circlearrowleft M_z = -125 \times 0.16$

$= -20$

$\vec{M_z} = \{-20\,\vec{k}\}$ N.m

$\vec{M_R} = \{-29.5\,\vec{j} - 20\,\vec{k}\}$ N.m